

Modeling and Predicting Stock Market Volatility using ARCH Model: Evidence from Bhutanese Stock Market

¹Sonam Darjay

¹Researcher and Founder, Center for Research, Thimphu,

Abstract

The stock market is widely known for its high degree of volatility, making it more challenging for individuals to accurately model and predict it in real-time. In an attempt to forecast the stock price and its volatility, investors, stock market analysts and academicians have been developing models for quite some time. One such model, the Autoregressive Conditional Heteroskedasticity (ARCH) model, developed by Engle (1982), has found wider applications in time series analysis where the central issue lies with the volatility clustering phenomenon.

This paper employs the ARCH model as an econometric model to forecast volatility of the share price of three companies listed on the ²Royal Securities Exchange of Bhutan, namely Sherza Ventures Limited (SVL), Bhutan Insurance Limited (BIL), and Bhutan National Bank Limited (BNB). For this study, we statistically tested the time series share price return data from January 2019 to December 2022 for stationarity using Augmented Dickey-Fuller unit root test and for conditional heteroscedasticity effect using the ARCH Lagrange Multiplier test before modelling the ARCH model.

To model the ARCH model, we selected the optimal order of the model, q , using the partial autocorrelation function of the squared residuals. The ARCH (1) model provided the best fit, resulting in accurate forecasts of the volatility of the stock price return. Overall, our ARCH (1) model performed very well with a mean absolute error and root mean squared error of 0.02628 and 0.03139, respectively, for the volatility of SVL return, 0.01523 and 0.01943 for BIL return, and 0.01395 and 0.01666 for BNB return.

The study is expected to benefit stock market investors for making a sound investment decision thorough their knowledge about the volatility of return. Also, it would enable the stock market regulators to develop effective regulations and policies to promote market efficiency and investor confidence. Additionally, the study may shed light to future researcher and academicians for potential future research on applications of the ARCH model beyond stock market.

Key words: ARCH, Augmented Dickey-Fuller test, ARCH-LM test, Volatility of Return, Modeling, Forecasting, Bhutan

²Royal Securities Exchange of Bhutan was established in August 1993

Introduction

The financial and economic time series data, such as stock prices, oil prices, and inflation, are considered highly volatile in nature, exhibiting a phenomenon known as volatility clustering (Engle, 1982; Gazda & Vyrost, 2003; Alberola, 2007; Zhang, Yao, He, & Ripple, 2019). Therefore, modeling and forecasting the volatility of the equity market has garnered significant interest due to its potential to provide valuable insights into stock market volatility, and its associated investment risks and return. One such modeling method is called the Box Jenkin's autoregressive integrated moving average (ARIMA) model, which is widely adopted for modeling the mean value of the variable in question (Gazda & Vyrost, 2003).

Modeling and forecasting agricultural commodity prices for example using ARIMA has become increasingly challenging due to the rapidly changing prices caused by actual and presumed changes in supply and demand conditions, exacerbated by weather-induced fluctuations in farm production (Lama, Jha, Paul, & Gurung, 2015). This highly volatile nature of time series data has made ARIMA model difficult to accurately forecast prices of commodities due to its limitations on assumptions of linearity and homoscedastic error variance. Therefore, Robert F. Engle introduced the ARCH model in 1982 as a non-linear model to deal with the heteroscedastic nature of the time series data.

The ARCH model has been widely utilized in numerous studies for forecasting the volatility of financial and economic markets. Engle (1983) applied the ARCH model to estimate the conditional mean and variance of inflation in the U.S using time series data. Engle, Ng, & Rothschild (1990) successfully priced Treasury bills using the FACTOR-ARCH model and demonstrated their stability over time. Degiannakis (2004) employed the ARCH model to generate more accurate volatility forecasts of stock returns. Alberola (2007) used the ARCH model to estimate the volatility of returns in the Spanish energy market, observing higher volatility compared to the gas and oil markets. Furthermore, Hu (2017) applied the ARCH-GARCH model to analyze stock market returns of China's listed real estate companies, revealing the presence of accumulation and memory effects, indicating the impact of past returns on current returns.

To study the volatility of stock market return, various econometric models, such as ARIMA, ARCH/Generalized ARCH (GARCH), Exponential GARCH, and Threshold GARCH models, have been adopted by researchers globally. However, such application of the econometric models in stock market remains limited in Bhutan. Therefore, this pilot study seeks to contribute to the existing gap by exploring the effectiveness of the ARCH model in modeling and predicting stock market volatility of three companies listed on the Royal Securities Exchange of Bhutan, namely SVL, BIL, and BNB.

The significance of the study lies in several aspects. Firstly, it will provide insights to the investors on the risk-return trade-off, helping them make better investment decisions. Secondly, it will offer some valuable information to market regulators regarding the impact of volatility on stock market performance, aiding in the formulation of effective regulations and policies. Finally, the study will pave way for future research on broader applications of ARCH and other econometric models beyond the stock market.

Methodology

ARCH Model

ARCH model was developed by Robert Engle in 1982 as a tool to capture the heteroscedastic characteristic or varying volatility of a financial time series data. The model assumes that the current volatility of the time series data is a function of the previous squared residuals and a constant term. The autoregressive model (AR) of order one with a stochastic error term is shown in equation (1) (Engle, 1982; Gökbulut & Pekkaya, 2014; Vasudevan & Vetrivel, 2016).

$$= a + bR_{t-1} + \varepsilon_t \quad \varepsilon_t | \psi_{t-1} \sim N(0, h_t), \quad (1)$$

Where, R_t is return at time t , a is the intercept term, b is coefficient of the lagged return R_{t-1} at time $t - 1$, ε_t represents the error term or disturbance term at time t , and ψ_{t-1} represents the information set available up to time $t - 1$. In the context of an ARCH model, the distribution of ε_t is assumed to be normal with mean zero and conditional variance h_t .

The equation for the conditional variance of the error term in a time series model called the autoregressive conditional heteroskedasticity (ARCH(q)) model is provided in equation (2).

$$h_t = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 \quad (2)$$

Where, h_t is the conditional variance of the error term at time t , which we are trying to model and forecast, α_0 is the intercept term, α_i is a coefficient of the i th lagged squared error term which is non-negative constant that determine the impact of past squared error terms on the current conditional variance, ε_{t-i}^2 represents the squared error term at time $t - i$, and q represents the maximum lag order. The squared errors capture the deviation of the actual value from the expected value based on the model's prediction.

The order of ARCH (1) model, where $q = 1$ has been identified by plotting the partial autocorrelation coefficient (PACF) of the squared residuals (Virginia, Ginting, & Elfaki, 2018).

Forecast Evaluation

There are various metrics for evaluating volatility forecast. In this study, to evaluate the forecasting performance of the ARCH (1) model, the mean absolute error (MAE) and, the root mean square error (RMSE) will be employed as per equation (3) and (4).

$$MAE = \frac{1}{n} \sum_{t=1}^n |y_t - \hat{y}_t| \quad (3)$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{t=1}^n (y_t - \hat{y}_t)^2} \quad (4)$$

Where n is the total number of observations, y_t is the actual value, and \hat{y}_t is the forecast value.

Statistical Testing

Augmented Dickey-Fuller Test

The Augmented Dickey-Fuller (ADF) unit root test is employed to test the null hypothesis for measuring presence of unit root in time series sample. For a return series R_t , the ADF test consists of a regression of the first difference of the series against the series lagged k times as provided in equation (5) (Dickey & Fuller, 1979; Gökbulut & Pekkaya, 2014; Virginia, Gunasekaran & Rajamohan, 2016; Ginting, & Elfaki, 2018).

$$\Delta Y_t = \mu + \beta Y_{t-1} + \varphi_1 \Delta Y_{t-1} + \varphi_2 \Delta Y_{t-2} + \dots + \varphi_p \Delta Y_{t-p} + \varepsilon_t \quad (5)$$

Where Y_t is the time series to be tested, μ is the intercept term, β is the coefficient of interest in the unit root test, $\varphi_1, \varphi_2, \dots, \varphi_p$ represent the coefficients of the lagged differenced variables $\Delta Y_{t-1}, \Delta Y_{t-2}, \dots, \Delta Y_{t-p}$, and ε_t represents the error term or residual at time t .

The ADF test is used to determine stationarity in a time series, where the null hypothesis is that the time series has a unit root, which implies non-stationarity, and the alternative hypothesis is that the time series is stationary. If the calculated ADF test statistic is less than the critical value at a given significance level, the null hypothesis is rejected, meaning stationarity of the time series sample. Conversely, if the calculated ADF test statistic is greater than the critical value at a given significance level, the null hypothesis is not rejected, implying non-stationarity.

ARCH LM Test

To test for the presence of conditional heteroscedasticity (ARCH effect) in a time series samples, researchers have employed the Lagrange multiplier (LM) test, which was introduced by Engle in 1982. The ARCH-LM test involves estimating an ARCH regression model and calculating the sum of squared residuals. Subsequently, the resulting ARCH-LM test statistic is compared to a critical value obtained from the chi-squared distribution, with degrees of freedom equal to the number of lags in the model. If the test statistic is greater than the critical value, the null hypothesis is rejected, indicating the presence of ARCH effect in the time series (Vasudevan & Vetrivel, 2016). Conversely, if the test statistic is less than or equal to the critical value, there is no significant evidence of ARCH effect, and the null hypothesis is not rejected.

Data

To model and predict the volatility of stock market return using ARCH (1) model, we downloaded the time series share price data from the official website of the Royal Securities Exchange of Bhutan (www.rsebl.org.bt). For this study, the secondary share price data for Sherza Ventures Limited (SVL) was considered from Jan 2020 to Dec 2022, and for Bhutan Insurance Limited (BIL) and Bhutan National Bank Limited (BNB), we used share price data from Jan 2019 to Dec 2022. The movement of the share prices for the three companies is shown in Figure 1 to Figure 3.

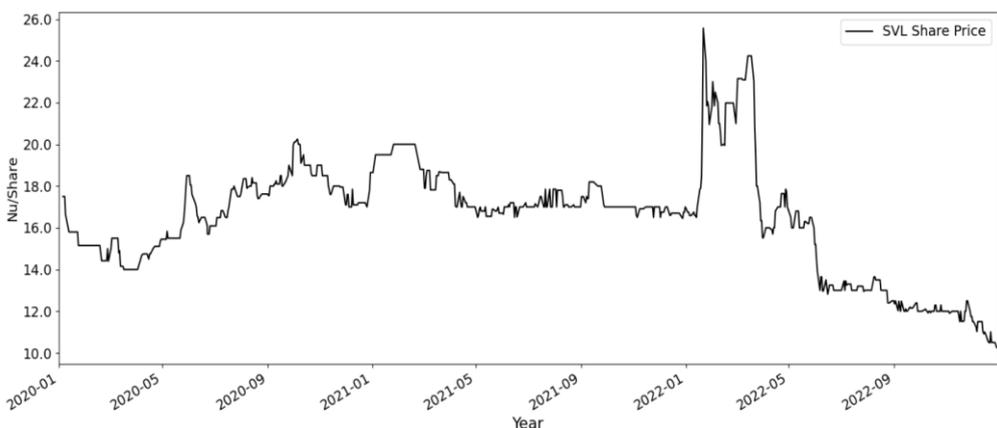


Figure 1 Share Price of SVL from Jan 2020 to Dec 2022

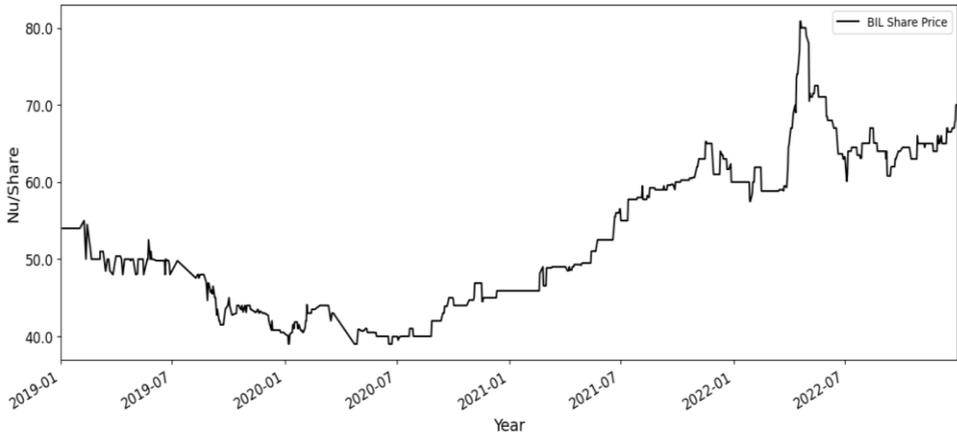


Figure 2 Share Price of BIL from Jan 2019 to Dec 2022

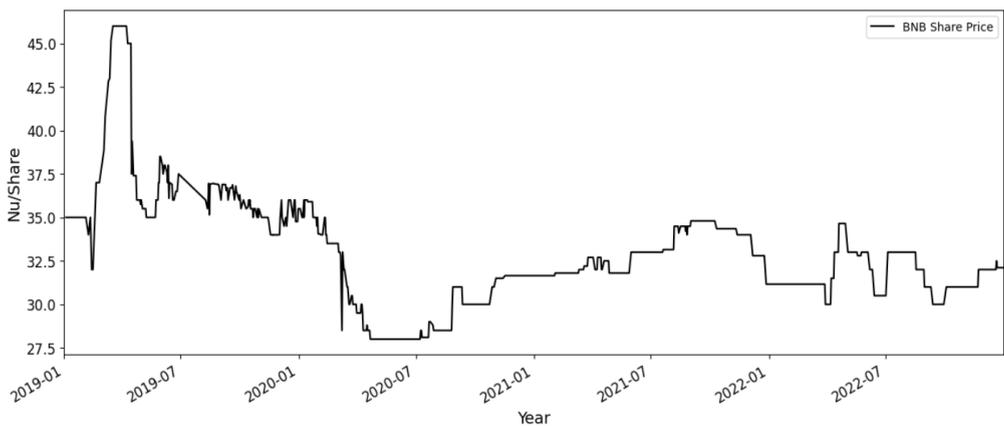


Figure 3 Share Price of BNB from Jan 2019 to Dec 2022

The daily return is calculated using the closing price data available from 2019 till 2022 as per equation (6).

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}} \times 100 \quad (6)$$

Where r_t is the daily return at time t , P_t is the opening share price at time t , and P_{t-1} is the closing share price at time $t - 1$. The movement of the daily returns for the three companies is provided in Figure 4 to Figure 6.

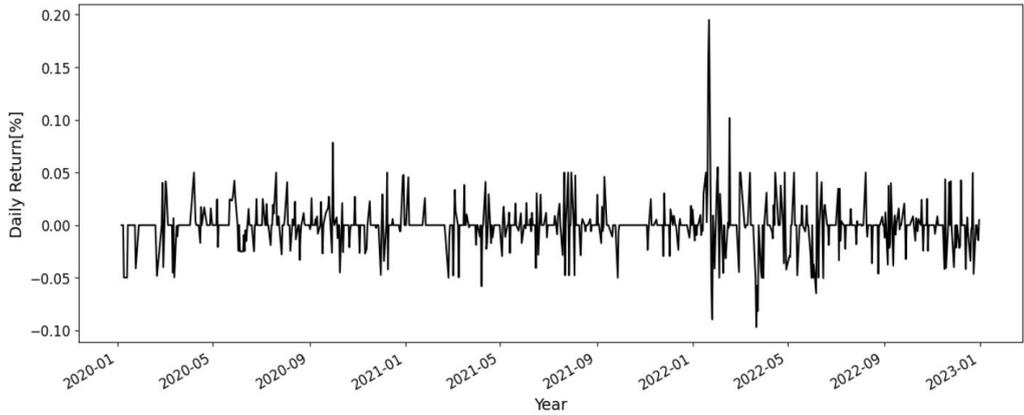


Figure 4 Volatility of the Daily Return for SVL from Jan 2020 to Dec 2022

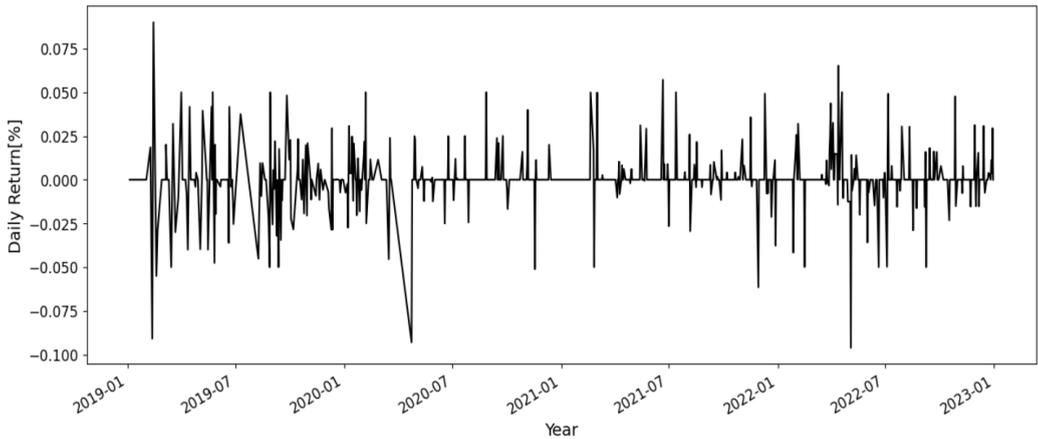


Figure 5 Volatility of the Daily Return for BIL from Jan 2019 to Dec 2022

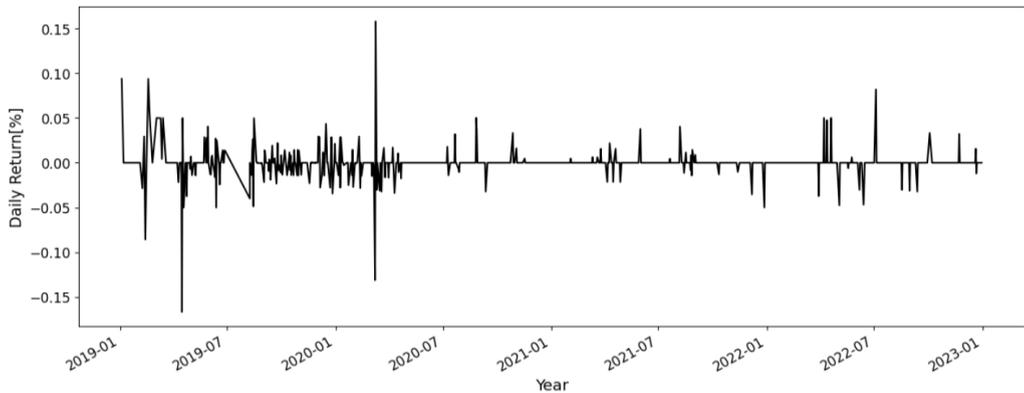


Figure 6 Volatility of the Daily Return for BNB from Jan 2019 to Dec 2022

Results & Discussion

Descriptive Statistics

Table 1

Descriptive Statistics of the Returns

Statistics	SVL Return	BIL Return	BNB Return
Mean	-0.000493	0.000426	0.000118
Maximum	0.194860	0.090000	0.157895
Minimum	-0.096830	-0.096154	-0.166667
Standard Deviation	0.022507	0.015093	0.015055
Skewness	1.211989	-0.467732	-0.249265
Kurtosis	12.890136	10.295969	42.441573
Jarque-Bera Test Statistics	5041.952967*	3656.06392*	67314.607286*
	(0.000)	(0.000)	(0.000)

Note: *– indicates significance at one per cent level.

Source: Descriptive statistics were computed using Python's pandas library

Table 1 provides summary of the descriptive statistics of the returns of three companies namely Sherza Ventures Limited, Bhutan Insurance Limited, and Bhutan National Bank Limited. The result suggests that the mean return for SVL is negative while positive for BIL and BNB. The standard deviation, which measures the variability of returns around the mean, is also highest for SVL followed by BIL and BNB. Jarque-Bera Test Statistics results with a low p-value (less than the significance level) indicates that the data is not normally distributed for all three different return data. The presence of non-normality in asset returns indicate that in order to accurately predict the volatility of asset return, a more robust and accurate model would be needed.

ADF Test Results

Table 2
ADF Test Statistics Results

Variable	p-value	Test Statistics	1% Critical Value	5% Critical Value	10% Critical Value
SVL Return	0.000000	-14.188588	-3.440	-2.866	-2.569
BIL Return	0.000000	-34.398530	-3.438	-2.865	-2.569
BNB Return	0.000000	-9.076169	-3.438	-2.865	-2.569

Source: Test was performed using Python's statsmodels library

Table 2 shows the results of the Augmented Dickey-Fuller (ADF) unit root test for three different returns. Having the test statistics for all three returns more negative than the critical values at 1%, 5%, and 10% significance level, rejects the null hypothesis of existence of unit root in the return series. Such ADF test result for stationarity was also observed by Islam (2013), and Vasudevan & Vetrivel (2016) during their studies on the volatility of stock market return. Therefore, the ADF test result ensures that we can adopt ARCH model to examine the dynamic behavior of volatility of the returns over time (Islam, 2013).

ARCH LM Test Results

Table 3
ARCH-LM Test Statistics Results

Dependent Variable of Model	Test Statistics	p-value	5% Critical Value
SVL Return	139.3343	0.0000	3.8415
BIL Return	33.7717	0.0000	3.8415
BNB Return	68.0645	0.0000	3.8415

Source: Test was performed using Python's arch package and Ordinary Least Squares regression

The results of the ARCH-LM test conducted on the return data for the three listed companies is provided in Table 3. The observed t-statistics surpasses the critical value at a 5% significance level, rejecting the null hypothesis. This indicates that the volatility of stock market returns exhibits a clustering phenomenon or ARCH effect, implying that it is not constant over time. To address this heteroscedasticity, an ARCH (1) model is employed for modeling and predicting the volatility of stock

market returns. By considering the ARCH effect, more accurate volatility forecasts can be obtained (Goudarzi, 2011).

Results of Estimated ARCH (1) Model

Table 4

Estimated Parameters of the ARCH (1) Model

$$R_t = a + bR_{t-1} + \varepsilon_t$$

$$h_t = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2$$

	<i>a</i>	<i>b</i>	α_0	α_i
SVL Return	-1.9862e-04	0.3620	3.1615e-04	0.3620
BIL Return	4.7187e-04	0.2000	1.1662e-04	0.2000
BNB Return	-4.7324e-04	0.2000	1.2445e-04	0.2000

ARCH (1)-LM Test: 0.05

Source: ARCH (1) model was estimated using Python’s arch model function in the arch library

Table 4 presents the estimates of the parameters for autoregressive model, and ARCH model for modeling and forecasting the volatility of the returns as per equation (1) and (2). The results of the ARCH (1)-LM test, conducted at a 5% significance level, provide evidence to reject the null hypothesis and support the existence of ARCH effect or time-varying volatility in all the returns. This finding suggests that the estimated parameters can now be used to model and forecast volatility of return using ARCH model. By doing so, the model can effectively capture and account for the non-normality and heteroscedastic characteristics that are present in the return data, allowing it to generate reasonable forecast (Vasudevan & Vetrivel, 2016).

Forecast Performance of ARCH Model

Table 5

Forecast Error Metric Results

	MAE	RMSE
SVL Return	0.02628	0.03139
BIL Return	0.01523	0.01943
BNB Return	0.01395	0.01666

Source: MAE and RMSE were calculated using Python’s NumPy library functions

Table 5 presents the results of the Mean Absolute Error (MAE) and Root Mean Square Error (RMSE). Among the three assets, BNB Return exhibits the lowest values for both MAE and RMSE, indicating that the ARCH models provide

relatively accurate volatility forecasts for BNB Return compared to SVL Return and BIL Return.

Actual Versus Forecast Volatility

Table 6
Actual Vs Forecast Volatility of Returns

	Volatility					
	Actual	Forecast	Actual	Forecast	Actual	Forecast
	SVL		BIL		BNB	
12/19/2022	0.01148	0.01832	0	0.01124	0	0.010689
12/20/2022	0.00023	0.02179	0	0.0107	0.00244	0.010683
12/21/2022	0	0.01601	0	0.01069	0.00144	0.012831
12/22/2022	0.02457	0.01592	0.00141	0.01068	0	0.011898
12/23/2022	0.02146	0.03354	0.00140	0.01079	0	0.010676
12/26/2022	0	0.02597	0	0.01078	0	0.010670
12/27/2022	0	0.01644	0.01253	0.01067	0	0.010664
12/28/2022	0.00091	0.01599	0.00136	0.01173	0	0.010658
12/29/2022	0.00208	0.01872	0.08651	0.01076	0	0.010652
12/30/2022	0.00024	0.01708	0	0.01683	0	0.010648

Source: Results were generated using Python's pandas library

Table 6 displays the actual and forecast volatility of returns for the three companies during a 10-day period in late December 2022. Notably, there is a significant variability between the actual and forecast volatility of returns for each financial asset. For example, on 12/19/2022, the actual volatility for SVL Return is 0.01148, while the forecast volatility is 0.01832. Conversely, on 12/29/2022, the actual volatility for BIL Return is 0.08651, whereas the forecast volatility is 0.01076. These discrepancies indicate that the ARCH model overestimated the volatility of SVL Return and underestimated the volatility of BIL Return. This highlights the limitations of the ARCH model in accurately predicting short-term volatility. Despite the model's capability in capturing the ARCH effect of time series data, the ARCH model might not fully account for unexpected events or sudden shifts in market dynamics, such as unexpected news announcements, economic factors, pandemics, and global economic shocks.

However, in addition to the ARCH models for modeling and forecasting stock market volatility, there are other advanced econometric models that can deal with issues related to unexpected events or sudden shifts in market dynamics. Such one model is the Exponential GARCH (EGARCH) model developed by Nelson

(1991), that allows for asymmetry in the volatility response to positive and negative shocks, capturing the notion that negative shocks might have a different impact on volatility compared to positive shocks (Gazda & Vyrost, 2003; Goudarzi, 2011; Islam, 2013). Also, Threshold GARCH (TGARCH) model introduced by Zakoian (1994) explains how volatility in financial markets responds to different situations, specifically focusing on the relationship between current volatility and past volatility levels (Gazda & Vyrost, 2003).

Conclusion

This study employed the ARCH (1) model to analyze and predict the volatility of stock market returns for three companies listed on Royal Securities Exchange of Bhutan; Sherza Ventures Limited (SVL), Bhutan Insurance Limited (BIL), and Bhutan National Bank Limited (BNB). The Augmented Dickey-Fuller (ADF) unit root test confirmed that the return data exhibited consistent statistical properties suitable for modeling volatility using the ARCH model. The ARCH-LM test further confirmed the presence of heteroscedastic characteristics in the return data.

Using the share price data from 2019 to 2022, our out-of-sample forecast using the ARCH (1) model demonstrated accurate predictions, with mean absolute errors ranging from 1.4% to 2.6% and root mean squared errors ranging from 1.7% to 3.1%. These results indicate the effectiveness of the ARCH model in capturing the time-varying volatility of stock market returns.

The findings of this study hold significance for investors and stock market analysts in providing them valuable insights for making informed investment decisions. Market regulators can also benefit from our study findings in developing appropriate regulations and policies for creating a conducive stock market environment. Furthermore, future research can build upon this study by exploring the application of ARCH models and other advanced statistical tools in various domains beyond the stock market.

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